

Chapter 3 Study Guide

Name: Key Date: _____
 ***** THIS IS NOT HOMEWORK *****

3.1	3.2	3.3	3.4	3.5
Definition of Exponentials Evaluating Exponentials Graphing Exponentials The Natural Base e	Definition of Logarithms Evaluating Logarithms with base a Graphing Logarithms with base a Evaluating Natural Logarithms	Change of Base Formula Product Property Quotient Property Power Property Expanding & Condensing	One-to-One vs. Inverse Properties Solving Exponential Equations Solving Logarithmic Equations Applications	Recognize the five most common types of models involving exponentials or logarithmic functions Use each model to solve real-life problems

Lesson 3.1 — Graph the exponential function by hand. Identify asymptotes and intercepts and determine whether the graph is increasing or decreasing.

1. $f(x) = 6^x$

x	y
-3	.005
-2	.03
-1	.17
0	1
1	6
2	36

HA: $y=0$
yint: $(0,1)$
increasing

2. $f(x) = 6^{-x}$

x	y
-2	36
-1	6
0	1
1	.17
2	.03

HA: $y=0$
yint: $(0,1)$
decreasing

3. $f(x) = 0.3^x$

x	y
-2	11.1
-1	3.3
0	1
1	.3
2	.09

HA: $y=0$
yint: $(0,1)$
decreasing

4. $f(x) = 0.3^{-x}$

x	y
-2	.09
-1	.3
0	1
1	3.3
2	11.1

HA: $y=0$
yint: $(0,1)$
increasing

Lesson 3.1 — Graph the natural base e exponential function. Identify asymptotes

5. $f(x) = e^{x-1}$

x	y
-1	.14
0	.37
1	1
2	2.7
3	7.4

HA: $y=0$

6. $f(x) = 2 + e^{-x}$

x	y
-2	14.8
-1	5.4
0	2
1	.7
2	.3

HA: $y=0$

Lesson 3.1 — Complete the table to determine the balance A for \$10,000 invested at a rate of 8% for t years, compounded continuously.

7.

t	1	20	50
A	\$10,832.87	\$49,530.32	\$54,598.50

$A = Pe^{rt}$

$A = 10000e^{.08(1)}$
 $A = 10000e^{.08(20)}$
 $A = 10000e^{.08(50)}$

Lesson 3.2 — Sketch the graph of the logarithmic function. Find the domain, VA, and x-intercept

8. $f(x) = -\log_2 x + 5$

$-(y-5) = \log_2 x$
 $2^{-y-5} = x$

x	y
1/32	-2
1/16	-1
1/8	0
1/4	1
1/2	2

D: $x \geq 0$
VA: $x=0$
xint: $(1,0)$

9. $f(x) = \log_5(x-3)$

$5^y = x-3$
 $5^y + 3 = x$

x	y
3.04	-2
3.2	-1
4	0
8	1

D: $x \geq 3$
VA: $x=3$
xint: $(4,0)$

Lesson 3.3 — Evaluate using the change-of-base formula. Do this for common and natural logs.

10. $\log_4 9$

$\frac{\log 9}{\log 4} = 1.585$ $\frac{\ln 9}{\ln 4} = 1.585$

11. $\log_{1/2} 8$

$\frac{\log 8}{\log 1/2} = -3$ $\frac{\ln 8}{\ln 1/2} = -3$

Lesson 3.3 — Expand the following.

12. $\ln \frac{xy^5}{\sqrt{z}}$

$\ln(xy^5) - \ln \sqrt{z}$
 $(\ln x + 5 \ln y) - \frac{1}{2} \ln z$

13. $\log_{10} \frac{5\sqrt{y}}{x^2}$

$\log_{10} 5 + \frac{1}{2} \log_{10} y - 2 \log_{10} x$

14. $\log_4 16xy^3$

$\log_4 16 + \log_4 x + \log_4 y^3$
 $\log_4 16 + \log_4 x + 3 \log_4 y$

15. $\ln \frac{x}{4}$

$\ln x - \ln 4$

Lesson 3.3 — Condense the following.

16. $\log_2 9 + \log_2 x$

$\log_2 9x$

17. $\frac{1}{2} \ln(2x-1) - 2 \ln(x+1)$

$\ln \sqrt{2x-1} - \ln(x+1)^2 = \ln \frac{\sqrt{2x-1}}{(x+1)^2}$

18. $\ln 3 + \frac{1}{3} \ln(4-x^2) - \ln x$

$\ln 3 + \ln \sqrt[3]{4-x^2} - \ln x$
 $\ln \frac{3\sqrt[3]{4-x^2}}{x}$

19. $3[\ln x - 2 \ln(x^2+1)] + 2 \ln 5$

$3 \ln \frac{x}{(x^2+1)^2} + 2 \ln 5$
 $\ln \left(\frac{x}{(x^2+1)^2}\right)^3 + \ln 5^2 = \ln 5^2 \left(\frac{x}{(x^2+1)^2}\right)^3$

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Lesson 3.4— Solve the following equations.

20. $\frac{2^{x+1}}{\log 2} = \frac{1}{\log 2^{16}}$
 $x+1 = \log_2 \frac{1}{16}$
 $x+1 = \frac{\log \frac{1}{16}}{\log 2}$
 $x = -5$

21. $\log_8 x = 4$
 $x = 4096$

22. $\ln(x-1) = 3$
 $x = 21.086$

23. $\frac{3e^{-5x}}{3} = \frac{132}{3}$
 $x = -.757$

24. $e^{2x} - 7e^x + 10 = 0$
 $(e^x - 5)(e^x - 2) = 0$
 $e^x - 5 = 0$ or $e^x - 2 = 0$
 $x = .693$
 $x = 1.609$

25. $\log_4(x-1) = \log_4(x-2) - \log_4(x+2)$
 $x-1 = \frac{(x-2)}{(x+2)} \rightarrow \frac{(x-1)(x+2)}{x^2+x-2} = \frac{x-2}{x-2}$

26. $\ln x - \ln 3 = 0$
 $\ln x = \ln 3$
 $x = 3$
 $x^2 = 0$
 $x = 0$

27. $2e^{x-3} - 1 = 4$
 $e^{x-3} = \frac{5}{2}$
 $x = 3.916$

28. $\log_{10}(-x-4) = 2$
 $100 = -x-4$
 $x = -104$

29. $\ln \sqrt{x+40} = 3$
 $e^{x+40} = (e^3)^2$
 $x = 363.429$

Lesson 3.5— Solve the following word problems.

30. The populations P (in thousands) of North Carolina from 1990 through 2008 can be modeled by $P = 6707.7e^{kt}$, where t is the year, with $t=0$ corresponding to 1990. In 2008, the population was about 9,222,000.

a. Find the value of k , and then $9222 = 6707.7e^{k(18)}$
 $k = .0177$

b. Use the result to predict the population in the year 2020. $\rightarrow t = 30$
 $y = 6707.7e^{.0177(30)}$
 $y = 11407.3$ thousands

31. The scores for a biology test follow a normal distribution modeled by $y = 0.0499e^{-(x-74)^2/128}$, where x is the test score and $40 \leq x \leq 100$.

- a. Use a graphing utility to graph the function (use the given domain and use .05 as YMAX and -.001 as YMIN)
- b. Use the graph to estimate the average test score. $x = 74$

32. The average number N of words per minute that the students in first grade class could read orally after t weeks of school is modeled by $N = \frac{62}{1+5.4e^{-.24t}}$. Find the numbers of weeks it took the class to read at average rates of

a. 40 words per minute $40 = \frac{62}{1+5.4e^{-.24t}}$
 $t = 9.5$

b. 60 words per minute $60 = \frac{62}{1+5.4e^{-.24t}}$
 $t = 21.19$

33. On the Richter scale, the magnitude R of an earthquake of intensity I is modeled by $R = \log_{10} \frac{I}{I_0}$ where $I_0 = 1$ is the minimum intensity used for comparison. Find the intensities I of the following earth quakes measuring R on the Richter scale.

a. $R = 7.1$
 $7.1 = \log_{10} I$
 $I = 10^{7.1}$

b. $R = 5.5$
 $5.5 = \log_{10} I$
 $I = 10^{5.5}$