9. The number of road construction projects that take place at any one time in a certain city follows a Poisson distribution with a mean of 7. Find the probability that five road construction projects are currently taking place in the city.

$x = 5$ $\lambda = 7$

$$P(5;7) = \frac{e^{-7} \cdot 7^5}{5!} = \boxed{0.1277}$$

10. Hospital records show that of patients suffering from a certain disease, 75% die of it. What is the probability that of 6 randomly selected patients with the disease, 2 will not recover?

Success = no recovery $n = 6$ $x = 2$ $p = .75$ $q = .25$

$$P(2) = {}_6C_2 \cdot (.75)^2 \cdot (.25)^{(6-2)} = \boxed{0.0330}$$

11. The probability that a person will have 0, 1, or 2 dental checkups per year is 0.2, 0.5, and 0.3, respectively. If seven people are picked at random, what is the probability that two will have no checkups, four will have one checkup, and one will have two checkups in the next year?

Events	Probability	x (# of the event)
E_1: No Checkup	P_1: 0.2	x_1: 2
E_2: 1 Checkup	P_2: 0.5	x_2: 4
E_3: 2 Checkups	P_3: 0.3	x_3: 1

$n = 7$

$$P(X) = \frac{7!}{(2!4!1!)} \cdot (.2)^2 \cdot (.5)^4 \cdot (.3)^1 = \boxed{0.0790}$$

12. A manufacturer of metal pistons finds that on the average, 12% of his pistons are rejected because they are either oversize or undersize. What is the probability that a batch of 10 pistons will contain <u>no more than 2 rejects</u>? $n = 10$ $x = 0, 1, 2$ $p = .12$ $q = .88$

$P(0) = {}_{10}C_0 \cdot (.12)^0 \cdot (.88)^{10} = 0.2785$

$P(1) = {}_{10}C_1 \cdot (.12)^1 \cdot (.88)^9 = 0.3798$

$P(2) = {}_{10}C_2 \cdot (.12)^2 \cdot (.88)^8 = +0.2330$

$= \boxed{0.8913}$

13. 800 batteries were checked last month after being manufactured by a company and found 90 defective batteries. What is the probability that this month in a random selection of batteries that 2 defective batteries will be found? This situation follows a Poisson distribution.

$x = 2$ $\lambda = \frac{90}{800} = 0.1125$

$$P(2;.1125) = \frac{e^{-.1125} \cdot .1125^2}{2!} = \boxed{0.0057}$$

14. In a Gallop Poll, 35% of college students stated they believe in ghosts. Find the probability that out of 16 college students <u>less than 4</u> said they believed in ghosts. $n = 16$ $x = 3, 2, 1, 0$ $p = .35$ $q = .65$

$P(3) = {}_{16}C_3 \cdot (.35)^3 \cdot (.65)^{13} = 0.0888$

$P(2) = {}_{16}C_2 \cdot (.35)^2 \cdot (.65)^{14} = 0.0353$

$P(1) = {}_{16}C_1 \cdot (.35)^1 \cdot (.65)^{15} = 0.0087$

$P(0) = {}_{16}C_0 \cdot (.35)^0 \cdot (.65)^{16} = +0.0010$

$= \boxed{0.1338}$

Quiz 8 Review Sheet

Name: Answers

Binomial, Multinomial, and Poisson Distributions

Vocabulary—*Use your notes to find the exact answer that fits each blank.*

1. The symbol used to denote a factorial is !.
2. A binomial experiment is a probability distribution that satisfies the following four requirements:
   a. Each trial can have only 2 outcomes or outcomes that can be reduced to two outcomes. These outcomes can be considered as either success or failure.
   b. There must be a fixed number of trials.
   c. The outcomes of each trial must be independent of each other
   d. The probability of a success must remain the same for each trial.
3. The formula used to find the probability of a binomial distribution is $P(X) = {}_nC_x \cdot p^x \cdot q^{n-x}$.
4. A multinomial experiment is a probability distribution that satisfies the following 3 requirements:
   a. Multinomial distributions are used if each trial in an experiment has more than two outcomes.
   b. This type of distribution can be used if the probabilities for each trial remain constant and the outcomes are independent for a fixed number of trials.
   c. The events must also be mutually exclusive.
5. The formula used to find the probability of a multinomial distribution is $\frac{n!}{(x_1!\,x_2!\ldots)} \cdot p_1^{x_1} \cdot p_2^{x_2} \ldots$
6. The symbol for the mean number of occurrences per unit is λ.
7. A Poisson distribution is used when the sample size, n, is large and the probability, p, is small and when independent variables occur over time.

Work Problems—*answer each question fully.*

Find the probability of each problem (*use the Binomial, Multinomial, or Poisson Distributions*).

8. Suppose a card is drawn randomly from an ordinary deck of playing cards, and then put back in the deck. This exercise is repeated five times. What is the probability of drawing 1 spade, 1 heart, 1 diamond, and 2 clubs?

Events	Probability	X (# of the event)
E_1: Spade	p_1: 1/4	x_1: 1
E_2: Heart	p_2: 1/4	x_2: 1
E_3: Diamond	p_3: 1/4	x_3: 1
E_4: Club	p_4: 1/4	x_4: 2

$n=5$

$$P(x) = \frac{5!}{(1!\,1!\,1!\,2!)} \cdot \left(\tfrac{1}{4}\right)^1 \cdot \left(\tfrac{1}{4}\right)^1 \cdot \left(\tfrac{1}{4}\right)^1 \cdot \left(\tfrac{1}{4}\right)^2 = \boxed{0.0586}$$

9. The number of road construction projects that take place at any one time in a certain city follows a Poisson distribution with a mean of 7. Find the probability that five road construction projects are currently taking place in the city.

$x = 5 \qquad \lambda = 7$

$$P(5;7) = \frac{e^{-7} \cdot 7^5}{5!} = \boxed{0.1277}$$

10. Hospital records show that of patients suffering from a certain disease, 75% die of it. What is the probability that of 6 randomly selected patients with the disease, 2 will not recover?

Success = no recovery $\quad n = 6 \quad x = 2 \quad p = .75 \quad q = .25$

$$P(2) = {}_6C_2 \cdot (.75)^2 \cdot (.25)^{(6-2)} = \boxed{0.0330}$$

11. The probability that a person will have 0, 1, or 2 dental checkups per year is 0.2, 0.5, and 0.3, respectively. If seven people are picked at random, what is the probability that two will have no checkups, four will have one checkup, and one will have two checkups in the next year?

Events	Probability	x (# of the event)
E_1: No Checkup	P_1: 0.2	x_1: 2
E_2: 1 Checkup	P_2: 0.5	x_2: 4
E_3: 2 Checkups	P_3: 0.3	x_3: 1

$n = 7$

$$P(X) = \frac{7!}{(2!\,4!\,1!)} \cdot (.2)^2 \cdot (.5)^4 \cdot (.3)^1 = \boxed{0.0790}$$

12. A manufacturer of metal pistons finds that on the average, 12% of his pistons are rejected because they are either oversize or undersize. What is the probability that a batch of 10 pistons will contain no more than 2 rejects? $\quad n = 10 \quad x = 0, 1, 2 \quad p = .12 \quad q = .88$

$P(0) = {}_{10}C_0 \cdot (.12)^0 \cdot (.88)^{10} = 0.2785$

$P(1) = {}_{10}C_1 \cdot (.12)^1 \cdot (.88)^9 = 0.3798$

$P(2) = {}_{10}C_2 \cdot (.12)^2 \cdot (.88)^8 = +0.2330$

$= \boxed{0.8913}$

13. 800 batteries were checked last month after being manufactured by a company and found 90 defective batteries. What is the probability that this month in a random selection of batteries that 2 defective batteries will be found? This situation follows a Poisson distribution.

$x = 2 \qquad \lambda = \frac{90}{800} = 0.1125$

$$P(2;.1125) = \frac{e^{-.1125} \cdot .1125^2}{2!} = \boxed{0.0057}$$

14. In a Gallop Poll, 35% of college students stated they believe in ghosts. Find the probability that out of 16 college students less than 4 said they believed in ghosts. $\quad n = 16 \quad x = 3, 2, 1, 0 \quad p = .35 \quad q = .65$

$P(3) = {}_{16}C_3 \cdot (.35)^3 \cdot (.65)^{13} = 0.0888$

$P(2) = {}_{16}C_2 \cdot (.35)^2 \cdot (.65)^{14} = 0.0353$

$P(1) = {}_{16}C_1 \cdot (.35)^1 \cdot (.65)^{15} = 0.0087$

$P(0) = {}_{16}C_0 \cdot (.35)^0 \cdot (.65)^{16} = +0.0010$

$= \boxed{0.1338}$