

Quiz 11 Review

Confidence Intervals & Critical Values

Name: Key
Date: _____ Period: _____

Vocabulary

1. The two values that you are confident the mean falls between is called the Confidence interval.
2. The Confidence level is the probability that the interval estimate will contain the parameter
3. The z-scores associated with the confidence level are called Critical values.
4. A point estimate is a specific numerical value estimate of the parameter
5. The three most common confidence levels are 90%, 95%, and 99%.
6. The total area in both tails of the normal distribution curve is denoted by α .
7. The t-Table contains critical values of the t distribution computed using the cumulative distribution function.
8. The degrees of freedom are the number of values that are free to vary after a sample statistic has been computed
9. The maximum error of estimate is the maximum difference between the estimate of the population mean and its actual value.

Work Problems

Pg. 61-62 - CRITICAL VALUES $Z_{\alpha/2}$

Find the critical values ($z_{\alpha/2}$) for:

(Show all steps!!!)

10. a 92% confidence interval

$$\begin{aligned} &\downarrow \\ &.92 \\ \alpha &= 1 - .92 = .08 \\ \frac{\alpha}{2} &= \frac{.08}{2} = .04 \rightarrow \underbrace{.0400}_{\text{area}} \end{aligned} \quad \boxed{Z_{\frac{\alpha}{2}} = \pm 1.75}$$

11. an 86% confidence interval

$$\begin{aligned} &\downarrow \\ &.86 \\ \alpha &= 1 - .86 = .14 \\ \frac{\alpha}{2} &= \frac{.14}{2} = .07 \rightarrow \underbrace{.0700}_{\text{area}} \end{aligned} \quad \boxed{Z_{\frac{\alpha}{2}} = \pm 1.48}$$

Pg. 63-66 - CRITICAL VALUES $t_{\alpha/2}$

Find the $t_{\alpha/2}$ values for each of the following.

(Show all steps!!!)

12. a 99% confidence interval when the sample size is 6.

$$\begin{aligned} n &= 6 \\ \text{d.f.} &= 6 - 1 = 5 \\ C &= 99\% \end{aligned} \quad \boxed{t_{\frac{\alpha}{2}} = 4.032}$$

13. a 95% confidence interval when the sample size is 15.

$$\begin{aligned} n &= 15 \\ \text{d.f.} &= 15 - 1 = 14 \\ C &= 95\% \end{aligned} \quad \boxed{t_{\frac{\alpha}{2}} = 2.145}$$

Pg. 61-62 - MAXIMUM ERROR OF ESTIMATE

14. A company took a sample of 75 customers in its contemporary apparel department. The population standard deviation is 7.5 years. Use a 93% confidence interval to find the maximum error of estimate for the age of customers

$\alpha = 1 - .93 = .07$
 $\frac{\alpha}{2} = .0350$ Area
 $Z_{\frac{\alpha}{2}} = \pm 1.81$
 $\sigma = 7.5$
 $n = 75$

$$E = Z_{\frac{\alpha}{2}} \left(\frac{\sigma}{\sqrt{n}} \right) = \pm 1.81 \left(\frac{7.5}{\sqrt{75}} \right) = \boxed{\pm 49.57}$$

15. The U.S. Bureau of Labor Statistics collects information on the ages of people in the civilian labor force and publishes the results in *Employment and Earnings*. A sample of 219 randomly chosen people was taken to determine the true mean age of all people in the civilian labor force. The sample mean was 37 years old. The population standard deviation is 12.1 years. Use a 99% confidence interval to find the maximum error of estimate for the age of the people.

$99\% \rightarrow Z_{\frac{\alpha}{2}} = \pm 2.58$
 $\sigma = 12.1$
 $n = 219$

$$E = Z_{\frac{\alpha}{2}} \left(\frac{\sigma}{\sqrt{n}} \right) = 2.58 \left(\frac{12.1}{\sqrt{219}} \right) = \boxed{\pm 2.11}$$

Pg. 61-62 - CONFIDENCE INTERVALS WHEN THE SAMPLE SIZE IS LARGE

16. The Library Journal is studying the mean annual subscription rates of law periodicals. A sample of 42 of this year's law periodicals yielded a sample mean of \$122.75 and a sample standard deviation of \$8.44. Determine a 95% confidence interval for this year's annual subscription rate, μ , for all law periodicals

$Z_{\frac{\alpha}{2}} = \pm 1.96$
 $n = 42$
 $\sigma = 8.44$
 $\bar{X} = 122.75$

$$122.75 - 1.96 \left(\frac{8.44}{\sqrt{42}} \right) < \mu < 122.75 + 1.96 \left(\frac{8.44}{\sqrt{42}} \right)$$

$$\boxed{\$120.20 < \mu < \$125.30}$$

17. The U.S. National Center for Health Statistics estimates mean weights of Americans by age, height, and sex. Forty U.S. women, 5ft 4in tall and age 18-24, are randomly selected and their weights, in pounds, are recorded. The sample mean is found to be 136.88 lbs. Assuming the population standard deviation of all such weights is 12.0 lbs, determine a 91% confidence interval for the mean weight, μ , of all U.S. women 5ft 4in tall and age 18-24.

$91\% \rightarrow \alpha = 1 - .91 = .09$
 $\frac{\alpha}{2} = .0450$ Area
 $Z_{\frac{\alpha}{2}} = \pm 1.70$
 $n = 40$
 $\sigma = 12$
 $\bar{X} = 136.88$

$$136.88 - 1.70 \left(\frac{12}{\sqrt{40}} \right) < \mu < 136.88 + 1.70 \left(\frac{12}{\sqrt{40}} \right)$$

$$\boxed{133.7 < \mu < 140.1}$$

Pg. 63-66 - CONFIDENCE INTERVALS WHEN THE SAMPLE SIZE IS SMALL

18. The A. C. Nielson Company publishes information on television viewing by Americans. A random sample of 20 U.S. households yields a sample mean of 7.71 hours per day and a standard deviation of 1.780 hours. Find a 95% confidence interval for the mean daily viewing time, μ , of all U.S. households.

$n = 20$
 $d.f. = 19$
 $C = 95\%$

$$t_{\frac{\alpha}{2}} = 2.093$$

$$7.71 - 2.093 \left(\frac{1.780}{\sqrt{20}} \right) < \mu < 7.71 + 2.093 \left(\frac{1.780}{\sqrt{20}} \right)$$

$$\boxed{6.877 < \mu < 8.543}$$

19. A random sample of 29 male wrestlers showed that their average weight was 256 pounds. The standard deviation of the sample was 8 pounds. Find the 99% confidence interval of the true mean.

$n = 29$
 $d.f. = 28$
 $C = 99\%$

$$t_{\frac{\alpha}{2}} = 2.763$$

$$256 - 2.763 \left(\frac{8}{\sqrt{29}} \right) < \mu < 256 + 2.763 \left(\frac{8}{\sqrt{29}} \right)$$

$$\boxed{251.895 < \mu < 260.105}$$