

FINAL EXAM STUDY GUIDE

CHAPTER 6—Radicals & Roots

6.2—Find the inverse of each function.

1. $f(x) = \frac{2x-1}{3}$ $f^{-1}(x) = \frac{3x+1}{2}$

3. $f(x) = \sqrt{x-2}$ $f^{-1}(x) = x^2 + 2$

5. $f(x) = 3x^2 - 9$ $f^{-1}(x) = \sqrt{\frac{x+9}{3}}$

2. $f(x) = 2x - 3$ $f^{-1}(x) = \frac{x+3}{2}$

4. $f(x) = x + 2$ $f^{-1}(x) = x - 2$

6. $f(x) = \frac{x+2}{3}$ $f^{-1}(x) = 3x - 2$

6.2—Determine whether each pair of functions are inverse functions. Write yes or no.

7. $f(x) = 3x - 1$

$g(x) = \frac{1}{3}x + \frac{1}{3}$ **Yes**

8. $f(x) = 2x + 5$

$g(x) = 5x + 2$ **No**

9. $f(x) = 8x - 12$

$g(x) = \frac{1}{8}x + 12$ **Yes**

6.4—Simplify.

10. $\sqrt[4]{256}$ **4**

12. $\sqrt[3]{64r^2w^{15}}$ **$4w^5\sqrt[3]{r^2}$**

14. $\sqrt[3]{27x^8y^{12}}$ **$3x^2y^4\sqrt[3]{x^2}$**

16. $\sqrt[3]{343d^6e^{14}}$ **$7d^2e^4\sqrt[3]{e^2}$**

11. $\sqrt[5]{243x^{10}}$ **$3x^2$**

13. $\sqrt{49m^2t^7}$ **$7mt^3\sqrt{t}$**

15. $\sqrt[3]{216p^3q^{11}}$ **$6pq^3\sqrt[3]{q^2}$**

17. $\sqrt[3]{64r^2w^{15}}$ **$4w^5\sqrt[3]{r^2}$**

6.4—Write each expression in radical form, or write each radical in exponential form.

18. $x^{\frac{1}{3}}$ **$\sqrt[3]{x}$**

20. $m^{\frac{4}{7}}$ **$\sqrt[7]{m^4}$**

22. $\sqrt[3]{27m^6n^4}$ **$27^{\frac{1}{3}}m^2n^{\frac{4}{3}}$**

24. $\sqrt[4]{f}$ **$f^{\frac{1}{4}}$**

19. $v^{\frac{2}{5}}$ **$\sqrt[5]{v^2}$**

21. $5^{\frac{1}{3}}$ **$\sqrt[3]{5}$**

23. $\sqrt[5]{2a^{10}b}$ **$2^{\frac{1}{5}}a^2b^{\frac{1}{5}}$**

25. $\sqrt[5]{d^{11}}$ **$d^{\frac{11}{5}}$**

6.4—Evaluate each expression.

26. $81^{\frac{1}{4}}$ **3**

27. $1024^{\frac{1}{5}}$ **4**

28. $8^{\frac{5}{3}}$ **32**

29. $64^{\frac{2}{3}}$ **16**

6.7—Solve each equation.

30. $2\sqrt{3x+4} + 1 = 15$ **$x = 15$**

32. $6 + \sqrt[3]{q-4} = 9$ **$q = 31$**

34. $\sqrt{6x-4} = \sqrt{2x+10}$ **$x = 3.5$**

31. $\sqrt[4]{y-9} + 4 = 0$ **No Solution**

33. $\sqrt{1-4t} - 8 = -6$ **$t = -0.75$**

35. $4\sqrt[3]{2x+11} - 2 = 10$ **$x = 8$**

6.5—Simplify by rationalizing.

36. $\frac{\sqrt{3}}{7}$ **$\frac{\sqrt{21}}{7}$**

37. $\frac{\sqrt[3]{2}}{3}$ **$\frac{\sqrt[3]{18}}{3}$**

38. $\frac{\sqrt{a^{12}}}{b^9}$ **$\frac{a^6\sqrt{b}}{b^5}$**

39. $\frac{\sqrt{\frac{2g^3}{5z}}}{5z}$ **$\frac{g\sqrt{10gz}}{5z}$**

6.5—Simplify by multiplying radicals.

40. $3\sqrt{5yz} \cdot 8\sqrt{10yz}$ **$120y^2z\sqrt{2}$**

42. $2\sqrt{32a^3b^5} \cdot \sqrt{8a^7b^2}$ **$32a^5b^3\sqrt{b}$**

44. $5\sqrt{2x^8y^3} \cdot 5\sqrt{2x^5y^4}$ **$50x^6y^3\sqrt{xy}$**

41. $16\sqrt[4]{3x^3y^2} \cdot \sqrt[4]{3xy^2}$ **$16xy^4\sqrt[4]{9}$**

43. $6\sqrt{3ab} \cdot 4\sqrt{24ab^3}$ **$144ab^2\sqrt{2}$**

45. $3\sqrt[3]{36xy} \cdot 2\sqrt[3]{6x^2y^5}$ **$36xy^2\sqrt[3]{2}$**

6.5—Simplify by adding & subtracting radicals.

46. $\sqrt{2} + \sqrt{8} + \sqrt{50}$ **$9\sqrt{2}$**

48. $2\sqrt{48} - \sqrt{75} - \sqrt{12}$ **$\sqrt{3}$**

50. $6\sqrt{20} + 8\sqrt{5} - 5\sqrt{45}$ **$17\sqrt{5}$**

47. $\sqrt{12} - 2\sqrt{3} + \sqrt{108}$ **$6\sqrt{3}$**

49. $8\sqrt{5} - \sqrt{45} - \sqrt{80}$ **$\sqrt{5}$**

51. $8\sqrt{48} - 6\sqrt{75} + 7\sqrt{80}$

$2\sqrt{3} + 28\sqrt{5}$

6.5—Simplify by BOX-ing radicals.

52. $(2 + \sqrt{3})(6 - \sqrt{2})$ **$12 + 6\sqrt{3} - 2\sqrt{2} + \sqrt{6}$**

54. $(3 - \sqrt{7})(5 + \sqrt{2})$ **$15 - 5\sqrt{7} + 3\sqrt{2} - \sqrt{14}$**

56. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$ **$5 - 2\sqrt{3} - \sqrt{30} + \sqrt{10}$**

53. $(1 - \sqrt{5})(1 + \sqrt{5})$ **-4**

55. $(\sqrt{2} - \sqrt{6})(\sqrt{2} - \sqrt{6})$ **$10 - 4\sqrt{3}$**

57. $(1 + \sqrt{6})(5 - \sqrt{7})$

$5 - \sqrt{7} + 5\sqrt{10} - \sqrt{42}$

6.5—Simplify by using the conjugate.

58. $\frac{3}{7-\sqrt{2}}$ **$\frac{21 + 3\sqrt{2}}{47}$**

60. $\frac{6}{\sqrt{2}-1}$ **$6\sqrt{2} + 12$**

62. $\frac{3+\sqrt{6}}{5-\sqrt{2}}$ **$\frac{15 + 5\sqrt{6} + 3\sqrt{2} + 2\sqrt{3}}{23}$**

59. $\frac{\sqrt{5}}{8-\sqrt{6}}$ **$\frac{8\sqrt{5} + \sqrt{30}}{58}$**

61. $\frac{3+\sqrt{2}}{2-\sqrt{2}}$ **$\frac{8 + 5\sqrt{2}}{2}$**

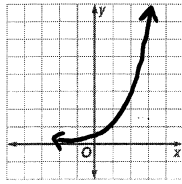
63. $\frac{5+\sqrt{3}}{4+\sqrt{3}}$ **$\frac{17-\sqrt{3}}{13}$**

CHAPTER 7—Exponentials & Logarithms

7.1—Make a table of values and graph the functions. State whether the graph is growth or exponential decay.

1. $y = \frac{1}{3}(3)^x$

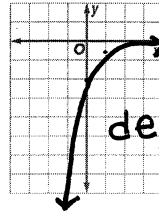
x	y
-1	.11
0	.33
1	1
2	3



growth

2. $y = -2\left(\frac{1}{4}\right)^x$

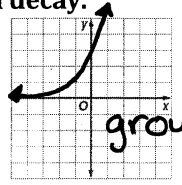
x	y
-1	-8
0	-2
1	-0.5
2	-0.13



decay

3. $y = 3(3)^x$

x	y
-3	.11
-2	.33
-1	1
0	3



growth

7.2—Solve each equation.

4. $9^{3c+1} = 27^{3c-1}$ $c = \frac{5}{3}$

5. $5^{x-6} = 125$ $x = 9$

6. $49^{x+5} = 7^{8x-6}$ $x = \frac{16}{6}$

7.3—Write each equation in exponential form.

7. $\log_{32} 8 = \frac{3}{5}$ $32^{\frac{3}{5}} = 8$

8. $\log_2 64 = 6$ $2^6 = 64$

9. $\log_6 216 = 3$
 $6^3 = 216$

7.3—Write each equation in logarithmic form.

10. $4^6 = 4096$ $\log_4 4096 = 6$

11. $2^8 = 256$ $\log_2 256 = 8$

12. $25^{\frac{3}{2}} = 125$ $\log_{25} 125 = \frac{3}{2}$

7.3—Evaluate each expression.

13. $\log_{121} 11$ $x = \frac{1}{2}$

14. $\log_{32} 2$ $x = \frac{1}{5}$

15. $\log_8 64$ $x = 2$

7.3—Convert to an exponential. Complete a table. Graph each function.

16. $y = \log_6 x$

$6^y = x$

x	y
.03	-2
.17	-1
1	0
6	1
36	2
216	3

17. $y = \log_{\frac{1}{4}} x$

$\frac{1}{4}^y = x$

x	y
16	-2
4	-1
1	0
.25	1
.01	2
1/64	3

7.4-7.5—Solve each logarithmic equation.

18. $\log_3 (4x - 17) = 5$ $x = 65$

20. $\log_{13} (x^2 - 4) = \log_{13} 3x$ $x = -1, x = 4$

19. $\log_9 x = \frac{5}{2}$ $x = 243$

21. $\log_3 (x - 5) = \log_3 (3x - 25)$ $x = 10$

7.4-7.5—Solve the following equations using the Product, Quotient, and Power Properties.

22. $\log_2 x - \log_2 5 = \log_2 4$ $x = 20$

25. $\log_6 2c + \log_6 8 = \log_6 80$ $c = 5$

23. $\log_{10} (x + 3) + \log_{10} x = \log_{10} 4$ $x = -4, 1$

26. $\log_{10} (c - 1) + \log_{10} c = \log_{10} (c - 1)$ $c = 1$

24. $\log_{10} 4 + \log_{10} w = 2$ $w = \frac{1}{2}$

27. $\log_{10} u = \frac{3}{2} \log_{10} 4$ $u = 8$

7.6—Use a calculator to evaluate each expression to the nearest ten-thousandth.

28. $\log 18$ 1.2553

29. $\log 39$ 1.5911

30. $\log 120$ 2.0792

7.6—Solve each equation. Round to the nearest ten-thousandth.

31. $4^{3x} = 12$ $x = .5975$

33. $5^{4x-2} = 120$ $x = 1.2437$

35. $2.4^{x+4} = 30$ $x = -.1150$

32. $6^{x+2} = 18$ $x = -.3869$

34. $7^{3x-1} = 21$ $x = .8549$

36. $6.5^{2x} = 200$ $x = 1.4153$

7.6—Express each logarithm in terms of common logarithms (Change of Base). Round to the nearest ten-thousandth.

37. $\log_3 16$ $\frac{\log 16}{\log 3} = 2.5237$

38. $\log_2 40$ $\frac{\log 40}{\log 2} = 5.3219$

39. $\log_5 35$ $\frac{\log 35}{\log 5} = 2.209$

7.7—Write an equivalent exponential or logarithmic equation.

40. $e^{15} = x$

$\ln x = 15$

41. $e^{3x} = 45$

$\ln 45 = 3x$

42. $\ln 20 = x$

$20 = e^x$

43. $\ln x = 8$

$x = e^8$

7.7—Solve each equation. Round to the nearest ten-thousandth.

44. $e^{4x} = 120$ $x = 1.969$
 45. $e^{x-2} + 4 = 21$ $x = 4.8332$
 46. $\ln(x+3) - 5 = -2$ $x = 17.0855$

47. $e^{-8x} = 50$ $x = -0.4890$
 48. $e^{4x-1} - 3 = 12$ $x = 0.9270$
 49. $\ln(5x+3) = 3.6$ $x = 6.7196$

$x = 0.5404$
 50. $6 + 3e^{x+1} + 1 = 21$
 51. $\ln(2x-5) = 8$ $x = 1492.9790$
 52. $\ln 5x + \ln 3x = 9$ $x = 23.2423$

7.8—POPULATION GROWTH:

In 2000, the world population was 6.124 billion people. In 2005, it was 6.515 billion.

53. Determine the value of k , the world's relative rate of growth. $k = 0.0125$
 54. When will the world's population reach 7.5 billion people? $t = 16.3$

For a certain strain of bacteria, k is 0.728 when t is measured in days.

55. Using the formula $y = ae^{kt}$, how long will it take 10 bacteria to increase to 675 bacteria? $t = 5.8$

7.8—LOGISTIC GROWTH:

The population of a certain species of fish in a lake after t years is given by $P(t) = \frac{1880}{1 + 1.42e^{-0.037t}}$.

56. When will the population reach 1600? $t = 56.6$

Assume the population of gnats in a specific habitat follows the function $P(t) = \frac{17,000}{(1 + 15e^{-0.0082t})}$.

57. When does the population reach 15,000? $t = 576$

7.8—EXPONENTIAL DECAY:

58. In 5 years, radioactivity reduces the mass of a 100-gram sample of an element to 75 grams. Find the constant k for this element for t in years using the equation, $f(x) = ae^{-kt}$. $k = 0.0575$

59. Radioactivity reduces the mass of a 100-gram sample of an element to 75 grams. If $k = .024$ for this element how many years, t would it take? Use the formula, $f(x) = ae^{-kt}$. $k = -0.000048$

CHAPTER 8—Rational Expressions

8.1—Simplify the following expressions.

1. $\frac{21x^3y}{14x^2y^2} \cdot \frac{3x}{2y}$

3. $\frac{24a^5b^2}{32a^4b^4} \cdot \frac{3a}{4b^2}$

5. $\frac{x^2+x-6}{x^2-6x-27} \cdot \frac{x-2}{x-9}$

2. $\frac{12x^3y^2z}{3x^2y^2z^4} \cdot \frac{4x}{z^3}$

4. $\frac{m^2-6m+5}{m^2-3m-10} \cdot \frac{m-1}{m+2}$

6. $\frac{3a^2-24a}{3a^2+12a} \cdot \frac{a-8}{a+4}$

8.1—Multiply and then simplify the following expressions.

7. $\frac{3m}{2f} \cdot \frac{f^3}{6} \cdot \frac{mf^2}{4}$

9. $\frac{5r^2}{r^2-4} \cdot \frac{r+2}{10r^5} \cdot \frac{1}{2r^4-4r^3}$

11. $\frac{w^2-5w-24}{w+1} \cdot \frac{w^2-6w-7}{w+3}$

8. $\frac{3r^2n^3}{5t^4} \cdot \frac{20t^2}{9r^3n} \cdot \frac{4n^2}{3rt^2}$

10. $\frac{c(c-3)}{c^2-25} \cdot \frac{c^2+4c-5}{c^2-4c+3} \cdot \frac{c}{c-5}$

w^2+w-56

8.1—Divide and then simplify the following expressions.

12. $\frac{80y^4}{49z^5v^7} \div \frac{25y^5}{14z^{12}v^5} \cdot \frac{32z^7}{35v^2}$

14. $\frac{q^2+2q}{6q} \div \frac{q^2-4}{3q^2} \cdot \frac{q^2}{2q-4}$

16. $\frac{3x+6}{x^2-9} \div \frac{6x^2+12x}{4x+12}$

13. $\frac{3xy^3}{5a^2} \div \frac{6x^2y}{a^3} \cdot \frac{ay^2}{10x}$

15. $\frac{x^2+8x+16}{2x-2} \div \frac{x^2+2x-8}{x-1} \cdot \frac{x+4}{2x-4}$

$\frac{2}{x^2-3x}$

8.1—Divide and then simplify the following COMPLEX fractions.

17. $\frac{\frac{c^2y}{2d^2}}{\frac{-c^6}{5d}} \cdot \frac{5y}{-2c^4d}$

18. $\frac{\frac{n+3}{n}}{\frac{n^2+8n+15}{n^4}} \cdot \frac{n^3}{n+5}$

19. $\frac{\frac{3p^2-3p}{4p+4}}{\frac{6p^2-6p}{p^2+p}} \cdot \frac{p}{8}$

8.2—Find the LCM of each set of polynomials.

20. a^2b^3c and abc^4 $a^2b^3c^4$

22. $x^3+3x^2-10x-24$
 x^2-x-6 and x^2+6x+8

24. $5a^2b^3c^2$, $10abc^5$, and $3a^3$

21. x^2y and xy^3 x^2y^3

23. $g-1$ and g^2+3g-4

25. $6p^2q^3r$, $4pq^4r^2$, and $3p^3r^4$

g^2+3g-4

$12p^3q^4r^4$

8.2—Simplify each MONOMIAL or POLYNOMIAL expression by adding or subtracting.

$$26. \frac{3y}{2x^3} + \frac{5z}{8xy^2} \quad \frac{12y^3 + 5x^2z}{8x^3y^2}$$

$$28. \frac{5}{6ab} - \frac{7}{8a} \quad \frac{20 - 21b}{24ab}$$

$$30. \frac{6}{2xy} + \frac{x^2}{3y} \quad \frac{18 + 3x^3}{6xy}$$

$$27. \frac{3}{mp} - \frac{4}{5m} \quad \frac{15 - 4p}{5mp}$$

$$29. \frac{5}{12x^4y} - \frac{1}{5x^2y^3} \quad \frac{25y - 12x^2}{60x^4y^2}$$

$$31. \frac{4m}{3mn} + 2 \quad \frac{4 + 6n}{3n}$$

$$32. \frac{16}{x^2 - 16} + \frac{2}{x + 4} \quad \frac{2}{x - 4}$$

$$34. \frac{3x + 3}{x^2 + 2x - 3} + \frac{x + 1}{x - 1} \quad \frac{x + 6}{x + 3}$$

$$36. \frac{2x}{x^2 - 5x + 6} - \frac{5}{x + 2}$$

$$33. \frac{2p - 3}{p^2 - 5p + 6} - \frac{5}{p^2 - 9} \quad \frac{2p^2 - 2p + 1}{p^3 - 2p^2 - 9p + 18}$$

$$35. \frac{5}{2x - 12} - \frac{20}{x^2 - 4x - 12} \quad \frac{5}{2x + 4}$$

$$37. \frac{y - 5}{y^2 - 3y - 10} + \frac{y}{y^2 + y - 2}$$

8.2—Simplify each COMPLEX expression by adding or subtracting.

$$38. \frac{\frac{2}{x} - 1}{\frac{1}{y} - \frac{3}{x}} \quad \frac{2y - xy}{x - 3y}$$

$$40. \frac{\frac{3}{x} + \frac{2}{y}}{1 + \frac{4}{y}} \quad \frac{3y + 2x}{xy + 4x}$$

$$42. \frac{\frac{2}{b} + \frac{5}{a}}{\frac{3}{a} - \frac{8}{b}} \quad \frac{2a + 5b}{3b - 8a}$$

$$39. \frac{\frac{c}{d} - \frac{d}{c}}{\frac{d}{c} + 2} \quad \frac{c^2 - d^2}{d^2 + 2cd}$$

$$41. \frac{\frac{2}{1} - \frac{x}{y}}{\frac{1}{2} + \frac{y}{x}} \quad \frac{4xy - 2x^2}{xy + 2y^2}$$

$$43. \frac{\frac{3}{x} + \frac{2}{y}}{1 + \frac{4}{y}} \quad \frac{3y + 2x}{xy + 4x}$$

8.6—Solve the following rational equations.

$$44. \frac{9}{10} + \frac{2}{x + 1} = \frac{2}{5} \quad x = -5 \checkmark$$

$$47. \frac{4t - 3}{5} - \frac{4 - 2t}{3} = 1 \quad t = 2 \checkmark$$

$$50. \frac{x}{x - 2} + \frac{4}{x - 2} = 10 \quad x = \frac{24}{9} \checkmark$$

$$45. \frac{2y}{3} - \frac{y + 3}{6} = 2 \quad y = 3 \checkmark$$

$$48. \frac{4}{x - 1} = \frac{x + 1}{12} \quad x = \pm 7 \checkmark$$

$$51. \frac{12p + 19}{p^2 + 7p + 12} - \frac{3}{p + 3} = \frac{5}{p + 4} \quad p = 2 \checkmark$$

$$46. \frac{3m + 2}{5m} + \frac{2m - 1}{2m} = 4 \quad m = -\frac{1}{24} \checkmark$$

$$49. \frac{2x + 1}{3} - \frac{x - 5}{4} = \frac{1}{2} \quad x = -\frac{13}{5} \checkmark$$

$$52. \frac{2f}{f^2 - 4} + \frac{1}{f - 2} = \frac{2}{f + 2} \quad f = -6 \checkmark$$

CHAPTER 10—Sequences and Series

10.1—Determine whether each sequence is arithmetic, geometric, or neither. Explain your reasoning.

1. 12, 36, 108, 324, ... *Geometric, The common ratio is three*

2. 6, 9, 14, 21, ... *Neither, there is no common ratio or difference*

3. $\frac{5}{2}, 3, \frac{7}{2}, 4, \dots$ *Arithmetic, the common difference is $\frac{1}{2}$.*

10.1—Find the next four terms in each sequence.

4. $\frac{2}{3}, -\frac{1}{3}, -\frac{4}{3}, \dots$ $-\frac{7}{3}, -\frac{10}{3}, -\frac{13}{3}, -\frac{16}{3}$

5. $\frac{1}{3}, 1, 3, 9, \dots$ 27, 81, 243, 729

6. 0.125, -0.5, 2, ... -8, 32, -128, 512

10.2—Find the sum of each series.

7. $\sum_{l=1}^{16} (4l - 2)$ 512

8. $\sum_{b=4}^{13} (4b + 1)$ 350

9. $\sum_{b=1}^8 (-3)(-2)^{b-1}$ 255

10.2—Write an equation for the nth term of each arithmetic sequence.

10. $a_7 = 21, d = 5$
 $a_n = 5n - 14$

11. 31, 17, 3, ...
 $a_n = -14n + 45$

12. $a_6 = 22, d = 9$
 $a_n = 9n - 32$

10.3—Find the geometric means of each sequence.

13. 810, , , 10
290, 90, 30 or
-290, 90, -30

14. 640, , , 2.5
160, 40, 10 or
-160, 40, -10

15. $\frac{729}{64}, , , \frac{324}{9}$
 $\frac{243}{16}, \frac{81}{4}, 27$ or
 $-\frac{243}{16}, \frac{81}{4}, -27$

10.3—Solve the word problem.

16. When an object is in free fall and air resistance is ignored, it falls 16 feet in the first second, an additional 48 feet during the next second, and 80 feet during the third second. How many feet will the object fall in 10 seconds?

1600 feet